Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands
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Abstract

The three-dimensional complexity of the attenuation structure below the Japan Islands is examined through tomographic inversion of the vertical components of ground velocity amplitudes. The attenuation structure should exhibit the same complexity as the velocity structure, as both properties are related to the elasticity (or inelasticity) of the medium. Using reported data for the period from January 1994 to December 2000, the three-dimensional seismic attenuation structure is determined in terms of the indices $Q_s$ and $Q_v$ for two frequency bands (5 and 2 Hz). Clear low-$Q$ zones are resolved beneath the volcanic front in northeastern Japan, accompanied by distinct high-$Q$ areas to the east of the front. A low-$Q$ area is also identified at a depth of 40 km below the Kanto region in central Japan, while a high-$Q$ area is distributed along the upper boundary of the Philippine Sea slab. Using this $Q$ distribution, ground motion amplitudes successfully reproduce the observations for a deep earthquake beneath the northeastern Kanto region.

Key words: Attenuation structure, $Q$, Amplitude Data, Tomography

1. Introduction

The seismic attenuation structure beneath the Japanese islands should be three-dimensionally complex to a similar degree as the velocity structure. For example, anomalous distributions of ground motion amplitudes are often observed, differing considerably from a circular distribution about an epicenter. By analogy to the velocity structure, high attenuation can be expected under volcanic fronts, while low attenuation should prevail along the subducting oceanic plates. However, the similarity with the velocity structure is unlikely to be seen in other parts of the Japanese islands because seismic attenuation implies inelasticity or scattering, whereas seismic velocity represents elastic properties. The attenuation of seismic waves has an important bearing on the physical states of the Earth's interior, including the temperature, degree of partial melting, and distribution of scattering matter. A precise estimate of the seismic attenuation leads to a better estimate of the strength of an earthquake source, in turn allowing for proper scaling. Information on seismic attenuation is also important in the simulation of strong ground motions.

In this study, tomographic inversions are performed for the three-dimensional (3D) attenuation structure beneath the Japanese islands. The seismic attenuation is usually specified by a quality factor $Q$, and several approaches have been proposed to estimate the $Q$ structure beneath the Japanese islands. These existing approaches can be divided into two groups according to the datasets used; spectral amplitudes, or seismic intensity data observed and compiled by the Japan Meteorology Agency (JMA).

Umino and Hasegawa (1984) inverted spectral ratios of P waves to S waves for determination of the 3D $Q_s$ structure in the Tohoku region assuming a constant $Q_p/Q_s$ ratio and identical source spectra for the P and S waves. Sekiguchi (1991) also investigated the $Q_s$ structure beneath the Kanto-Tohoku district from P- and S-wave spectral ratios with the same assumptions. Tsumura et al. (1995, 2000) obtained a fine-scale model of the $Q_s$ structure in the Tohoku region using P-wave spectra without assuming a constant $Q_p/Q_s$ ratio, instead introducing the $Qs$ model (Aki, 1967) for the source spectra.

Hashida and Shimazaki (1984) formulated the inversion of seismic intensities using JMA data, and later (Hashida and Shimazaki, 1985, 1987) estimated the 3D $Q_s$ structure beneath the Tohoku and Kanto regions in northeastern Japan. It was assumed in those studies that S waves with isotropic radiation represented the main contribution to seismic intensities. Hashida (1989) extended the analysis to the entire Japan region, and was...
followed by a detailed study by Nakamura et al. (1994). Satake and Hashida (1989) applied the inverse method to intensity data in New Zealand, which is located proximal to a subduction zone similar to Japan.

As the creation of a dataset of spectral amplitudes is difficult and laborious, inversion has been carried out in only a limited number of areas. In contrast, large set of seismic intensities have already been compiled by the JMA, requiring little effort for utilization of those data. However, as the relationship between seismic intensities and ground motions is not clear, great uncertainties can be expected in the results of intensity inversion. JMA has also compiled amplitude data, which are available in the JMA catalogs. The data are provided as maximum amplitudes of actual ground motions observed by seismometers, and thus involve no ambiguity in terms of physical meaning. In this study, the tomographic inversion of amplitude data is formulated and applied to extensive JMA data to obtain detailed images of regional $Q_p$ and $Q_s$ structures beneath the Japanese islands.

In Chapter 2, the characteristics of datasets in the JMA catalogs are considered, and the use of ground motion amplitudes for tomographic inversion is discussed. The methodological details of the tomographic inversion are presented in Chapter 3, including observation equations, the least-squares algorithm, the model configuration, ray tracing, source definitions, and other settings. As the proposed tomographic inversion is non-linear, an initial estimate of the attenuation structure is required and smoothing constraints are introduced to stabilize the inversion as model regularization. In Chapter 4, the results obtained by tomographic inversion are presented, and the features of the data, such as low- and high-$Q$ zones are discussed. The descending slabs and the relationship between the derived source strengths and earthquake magnitudes are also considered. Checkerboard tests are conducted as part of this discussion to verify the resolution of the inversion of amplitude data. Finally, in Chapter 5, ground motion simulations are performed using the derived 3D attenuation structure and the results compared with the observed amplitude distributions.

2. Data

The data used for tomographic inversion of the 3D attenuation structure beneath the Japanese islands are selected from the Annual Seismological Bulletins of the JMA. In this chapter, the data, selection criteria and the observational instruments installed at stations operated by the JMA are described, and the method used to determine the maximum amplitudes of P or S waves is introduced. As the initial source strengths are calculated from the magnitudes determined by the JMA, the JMA magnitudes are also discussed.

2.1 JMA stations and instruments

JMA operated 359 stations across the Japanese islands prior to October 1997. The EMT and EMT76 seismometers originally installed at these stations were gradually replaced with new E93 seismometers. However, as the response curve of the E93 is almost the same as that of the EMT and EMT76, the differences between seismometers are not considered in this study. The seismometers consist of short-period electromagnetic sensors that record two or three components of ground velocity at 100 samples per second.

In October 1997, the JMA network was combined with stations operated by national universities and the National Research Institute for Earth Science and Disaster Prevention (NIED). The combined network includes more than 1000 stations, of which only 947 stations report amplitudes in the Annual Seismological Bulletins of the JMA (Fig. 1). These bulletins are collectively referred to here as the JMA catalog.

2.2 JMA catalog

Vertical amplitudes of ground velocities reported between January 1994 and December 2000 are used in this study. Horizontal amplitude data are not used due to the lack of reported data in the period before amalgamation of the networks in October 1997. At the time of amalgamation, the criterion applied to the reporting of amplitudes was changed such that the number of reports was limited to the 40 stations closest to the epicenter of each earthquake. However, restricting the data to those after October 1997 will cause most ray paths between hypocenters and reporting stations to be nearly vertical, resulting in a loss of tomographic resolution. The use of data recorded before and after October 1997 is therefore critical for the present inversions. Although the data accuracy could be improved by deriving the wave spectrum from each record, the processing
of so many records is prohibitive. It is therefore more useful to use the maximum amplitude data for calculation of attenuation inversion.

The seismic attenuation is represented by the indices $Q_p$ and $Q_s$, which are dependent on the frequencies of ground motion. The 3D structure is obtained for two frequency bands, 5 Hz (0.1-0.3 s) and 2 Hz (0.4-0.6 s), by selecting amplitudes of ground motions with periods within these frequency bands. This selection is carried out based on the following criteria:

1. Earthquakes are located within the study area.
2. Earthquakes at depth shallower than 10 km are omitted as potentially generating surface waves larger than the S waves.
3. Earthquakes have at least five amplitude reports within the frequency bands (three reports in southeastern Japan due to the fewer earthquakes in that area).
4. Maximum amplitudes are recorded within 2 s of the arrival of a P or S wave.

Amplitudes from 2328 and 3236 earthquakes are selected for P- and S-wave tomography in the 5 Hz band, respectively. The hypocenters of these earthquakes are plotted in Fig. 2 and Fig. 3. For the 2 Hz band, amplitudes from 1016 and 1446 earthquakes are used in P- and S-wave tomography, respectively. More deep earthquakes are available for P-wave tomography than for S-wave tomography due to the near-vertical orientation of most rays from deep earthquakes. In this situation, P waves represent the dominant contribution to the maximum amplitude of vertical ground motion.

2.3 Ground velocity amplitudes

The catalog entry for a magnitude 3.7 earthquake that occurred on September 20, 2000 is shown in Fig. 4. The first line indicates the hypocentral information, and the other lines show reports from stations, including the station name, station code, type of seismometer, and the on-set quality and arrival time for P waves and S waves. The right half of each line describes the amplitude information for NS, EW and vertical ground motions. The maximum amplitude, period of maximum amplitude motion, seconds of maximum amplitude relative to the P-wave arrival time, and the unit of amplitude are given in the amplitude information. JMA defines the maximum amplitude as the maximum peak-to-peak velocity amplitude within an entire trace. The period of maximum amplitude motion is calculated by doubling the interval between these peaks.

The earthquake occurred in eastern Yamanashi Prefecture. The seismograms observed at Hachioji (E.HCJ) and Oyama (E.OYM), available from J-arary, are plotted in Fig. 5. In the trace at Hachioji, the maximum peak-to-peak amplitude occurred immediately after the arrival of the P wave, with a 0.15 s interval between peaks. These findings are in agreement with the appearance time of less than 1 s after P-wave arrival and the period of 0.3 s in Fig. 4. The lower trace for the Oyama station shows that the maximum peak-to-peak amplitude occurred 17 ~ 18 s after the P-wave arrival, with a peak-to-peak interval of 0.2 s. This is also in agreement with the description in the corresponding line in Fig. 4 (E.OYM). As the S-wave arrives 17.52 s after P-wave arrival, the maximum amplitude appears...
Fig. 4  Entry in the Annual Seismological Bulletins of JMA for an earthquake on September 2, 2000.

Fig. 5  Vertical ground motions observed at the Hachioji (upper) and Oyama (lower) stations. Arrows denote arrivals of P and S waves. Maximum peak-to-peak amplitudes are shaded.

Fig. 6  Spectrum of ground motion around the amplitude maximum at Hachioji (E.HCJ). Period of maximum amplitude motion reported by JMA is shaded.

Fig. 7  Spectrum of ground motion around the amplitude maximum at Oyama (E.OYM). Period of maximum amplitude motion reported by JMA is shaded.
immediately upon arrival of the S wave. If the P and S waves are assumed to form wave trains, the amplitudes at Hachioji and Oyama can be considered to be the amplitudes of the P- and S-wave trains.

The periods of maximum amplitude motions were verified by generating Fourier spectra for the traces in the periods of the maximum amplitudes in Fig. 6 and Fig. 7. Based on this verification and the comparison above, amplitude data with appearance times within 2 s of P- or S-wave arrival are selected as P- and S-wave amplitudes. The data are then included in the datasets for tomographic inversions if the period falls into either of the frequency bands considered (5 Hz or 2 Hz).

2.4 JMA magnitudes

As the present tomographic inversions are non-linear, initial values are required for the source strengths of the earthquakes and the $O_Q$ and $Q_r$ structures. The initial source strengths are calculated from magnitudes determined by JMA, Kanbayashi and Ichikawa (1977) and Takeuchi (1983) proposed the formulas

$$M = \log_{10} A_c + 1.64 \log_{10} \Delta + 0.22$$

$$M = \log_{10} A_c + 1.64 \log_{10} \Delta + 0.44$$

for computing JMA magnitudes of small shallow earthquakes (depth $\leq$ 90 km) from amplitudes measured by short-period velocity seismometers. Here, $A_c$ is half the maximum peak-to-peak velocity amplitude of ground motion in milli-ke (10^{-3} cm/s), and $\Delta$ is the epicentral distance in kilometers. If $A_c$ is measured by a seismometer at the surface, Equation (2.1) is adopted. Equation (2.2) is employed for amplitudes observed in boreholes.

3. Method

3.1 Introduction

In this chapter, the methods used for tomographic inversion of attenuation structures are introduced. This attenuation tomography involves modeling the Earth’s structure, modeling earthquake sources, calculating ray paths and travel times, and solving a least-squares problem. The observation equations employed for attenuation tomography are linear equations but are solved iteratively in a similar manner to non-linear problems to account for the large observational errors expected in the amplitude data. Assumptions for modeling of earthquake sources are imposed on the strength and pattern of source radiation, and ray tracing and the calculation of travel times are then performed in a three-dimensionally complex velocity structure. Geometrical spreading factors for amplitude decay are calculated from the total length of the ray path, and the derivatives of amplitudes with respect to $Q_d$ or $Q_r$ are obtained from the lengths of ray segments. The velocity model and initial estimate of the $Q_d$ and $Q_r$ structures are also discussed in this chapter.

3.2 Observation equations

3.2.1 Convolution representation

Seismic amplitudes are controlled not only by the attenuation structure but also various effects such as source and site effects and geometrical spreading. These controls can be formulated by the convolution for the amplitude of the $j$th earthquake at the $j$th station as follows.

$$A_j(f) = S_j(f) \cdot G_j(f) \cdot R_j^{-1} \cdot B_0(f),$$

where $S_j$ and $G_j$ are the effects of the $j$th source and the site response at the $j$th station, $R_j^{-1}$ is the inverse of a ray length, which represents geometrical spreading, and $B_0(f)$ is the effect of the attenuation structure between the $j$th source and $j$th station. $B_0(f)$ is rewritten with $v^0$ as

$$B_0(f) = \exp(-\pi f v^0).$$

The natural logarithm of Equation (3.1) is then taken and the result linearized as

$$\ln A_j(f) = \ln S_j(f) + \ln G_j(f) - \ln R_j - \pi f v^0 \cdot.$$ (3.3)

Since the pioneering studies of Aki and Lee (1976) and Aki et al. (1977), block modeling has been adopted in many tomography studies (e.g., Hirahara, 1988; Hashida and Shimazaki, 1984; Nakamura et al., 1994). In this approach, the structure is divided into a set of boxes, within which the seismic velocity and attenuation are assumed to be constant. According to the definition of $Q_d$, if the $i$th ray travels through the blocks with $Q_{0i}$ ($k_i = 1, 2, ..., N_i$), we have

$$t_i^0 = \sum_{k_i=1}^{N_i} T_{kj} Q_{kj}^{-1} = \sum_{k_i=1}^{N_i} \frac{\Delta s_{kj}}{V_{kj}} Q_{kj}^{-1}.$$ (3.4)

The ray stays in the $i$th block for the duration $t_{ij}$, which can be calculated from the velocity $V_{ij}$ and length $\Delta s_{ij}$ of the ray segment in this block. However, ray tracing in block modeling is rather difficult due to the presence of singularities of velocity at block boundaries.

3.2.2 Grid modeling

Thurber (1983) developed an alternative approach to space modeling in which the 3D heterogeneous structure is represented by a 3D grid of nodes. Parameters (velocities or $Q$) vary continuously in all directions, with Lagrange interpolation among nodes. The parameter in this representation can be defined anywhere in the modeling space, removing any difficulties in ray tracing and allowing for the introduction of laterally varying discontinuities. Zhao et al. (1992) adopted this approach for travel-time tomography of data for the Tohoku region, northeastern Japan, and succeeded in deriving a fine-scale velocity structure for the subducting Pacific Plate and the mantle wedge using a pseudo-bending ray tracing technique (Um and Thurber, 1987) with irregular discontinuities. Negishi (1999) modified this grid modeling approach for global attenuation tomography of amplitude data provided by the International Seismological Center.

In the grid configuration, $Q^0$ on the ray path is calculated by interpolation of $Q^j$ at grid nodes surrounding the ray path.
Therefore, $Q^1$ at grid nodes is the only unknown parameter to be solved. If a given point $(\phi, \lambda, h)$, for latitude $\phi$, longitude $\lambda$, and depth $h$, is located in a cube spanning $(\Phi_i, \Phi_{i+1}, \lambda_i, \lambda_{i+1})$ and $(h_k, h_{k+1})$, $Q^1$ at the given point can be obtained by Lagrange interpolation of the 8 adjacent nodes, as follows:

$$Q^1(\phi, \lambda, h) = \sum_{i=1}^{8} Q^1(W_i)$$

$$= \sum_{i=1}^{8} \sum_{j=1}^{8} Q^1(\phi, \lambda, h)$$

$$= \left\{ \begin{array}{l} 1 - \frac{\phi - \phi_i}{\phi_{i+1} - \phi_i} \\ 1 - \frac{\lambda - \lambda_i}{\lambda_{i+1} - \lambda_i} \\ 1 - \frac{h - h_k}{h_{k+1} - h_k} \end{array} \right\}$$

(3.5)

where $W_i$ is the weighting function, $\phi_i$, $\lambda_i$, and $h_k$ are the coordinates of the 8 adjacent nodes, and $Q^1(\phi, \lambda, h)$ is the reciprocal of the quality factor at the $ijk$th grid node. The slowness at the position $(\phi, \lambda, h)$ is expressed by a similar equation:

$$V^{-1}(\phi, \lambda, h) = \sum_{i=1}^{8} V^{-1}(W_i)$$

$$= \sum_{i=1}^{8} \sum_{j=1}^{8} V^{-1}(\phi, \lambda, h)$$

$$= \left\{ \begin{array}{l} 1 - \frac{\phi - \phi_i}{\phi_{i+1} - \phi_i} \\ 1 - \frac{\lambda - \lambda_i}{\lambda_{i+1} - \lambda_i} \\ 1 - \frac{h - h_k}{h_{k+1} - h_k} \end{array} \right\}$$

(3.6)

A range of grid configurations can be employed for the velocity and attenuation models. Substituting Equation (3.5) into Equation (3.4) thus yields

$$t_s^i = \sum_{k=1}^{N_s} V_{s_k}^{-1} \Delta s_{k_i} = \left[ \sum_{k=1}^{N_s} Q_{s_k}^{-1} \right]_{s_k}$$

(3.7)

where $N$ is the number of segments in the ray path, and $W_i$ is the interpolation weight of the $k$th node for the $i$th segment. In this grid modeling, the observation equation (3.3) then becomes

$$\ln(A_{s_i} R_s) = \ln S_i + \ln G_s$$

$$-\pi f \sum_{k=1}^{N_s} V_{s_k}^{-1} \Delta s_{k_i} \sum_{k=1}^{N_s} Q_{s_k}^{-1} \omega$$

(3.8)

$$= \ln S_i + \ln G_s$$

$$-\pi f \sum_{k=1}^{N_s} V_{s_k}^{-1} \Delta s_{k_i} \sum_{k=1}^{N_s} \delta_{m,n} Q_{s_k}^{-1} \omega$$

(3.9)

where $M$ is the number of grid points in the entire model and $\delta$ is the Kronecker delta.

As the observation equation (3.8) looks complex but is in fact linear, it can be solved directly. However, such a direct solution is often subject to numerical instability or local minima, particularly in large-scale inversions of noisy data such as amplitudes. Therefore, Equation (3.8) is solved iteratively in the present simulation. Using the initial estimates in the previous iteration as references of $Q_{s_k}^{-1}$, Equation (3.8) can be modified as

$$\Delta \ln A_{s_i} = \Delta \ln S_i + \Delta \ln G_s$$

(3.10)

$$-\pi f \sum_{k=1}^{N_s} V_{s_k}^{-1} \Delta s_{k_i} \sum_{k=1}^{N_s} \delta_{m,n} Q_{s_k}^{-1} \omega$$

where $\Delta \ln S_i$ and $\Delta \ln G_s$ are the unknown perturbations of the logarithms of source and site effects from the reference effects, respectively. Since changes in $S_i$, $G_s$ or $Q_{s_k}^{-1}$ are assumed not to affect the ray paths, $\Delta \ln A_{s_i}$ is adopted as the logarithm for the ratio between the observed amplitudes and those calculated from the reference parameters.

In this formulation, the perturbations from the reference model, $\Delta \ln S_i$, $\Delta \ln G_s$ and $\Delta Q_{s_k}$, are unknown parameters in the least-squares inversion. These unknowns are collectively represented by the vector $m$, and Equation (3.10) is simply written as

$$d = G m$$

(3.11)

where $d$ consists of $\Delta \ln A_{s_i}$. The elements of $G$ for $\Delta \ln S_i$ and $\Delta \ln G_s$ are 1 or 0, and the other part of $G$ for $\Delta Q_{s_k}$ consists of $-\pi f V_{s_k}^{-1} \Delta s_{k_i} \delta_{m,n} \omega$.

### 3.2.3 Grid configuration

The study area is set to the region 30-46°N and 130-145°E. This area covers the four main islands of Japan, excluding the easternmost tip of Hokkaido and the westernmost tip of Kyushu. A grid with interval of 0.50 is applied to this region at depths of 10, 25, 40, 65, 90, 120, and 150 km, then every 30 km to a maximum depth of 360 km. The first two depths correspond to the upper and lower parts of the crust, and the remaining levels correspond to mantle. As shown in Fig. 8, the grid is extended to 28-48°N and 122-148°E to cope with large ray deviations, which may occur during the iteration.

### 3.3 Source effects

#### 3.3.1 $\omega^2$ model

The $\omega^2$ model of Aki (1967) is introduced to model the source effects $S$ as a function of frequency $f$. The sources are constrained by (Brune, 1970, 1971; Boore, 1983)

$$S_i = \frac{R_{s_i}^{\text{dev}}}{4\pi^2 f^2} \left[ \frac{2\pi f}{f_0 f_s} \right] M$$

(3.12)

where $M$, $f_0$, and $R_{s_i}^{\text{dev}}$ are the seismic moment, corner frequency and radiation coefficient of the $i$th earthquake, respectively. The S-wave velocity $V_s$ and density $\rho_s$ are measured in the source region of the earthquake. The corner frequency $f_s$ depends on
parameterized shooting method (Sekine and Koketsu, 2001), and the takeoff and azimuthal angles ($\theta$ and $\psi$) are calculated for all amplitude data used in the $Q$ tomography. If the mechanism of an earthquake is not determined by JMA, the mechanism is inferred as a strike $\phi$, dip $\delta$ and rake $\lambda$ from the initial motion polarities of P waves. The theoretical radiation patterns $r_j$ are then using the equation of Aki and Richards (1980):

$$R_p = \cos \lambda \sin \delta \sin^2 \theta \sin (2(\psi - \phi)$$

$$- \cos \lambda \cos \delta \sin 2\theta \cos (\psi - \phi)$$

$$+ \sin \lambda \sin 2\delta (\cos^2 \theta - \sin^2 \theta \sin^2 (\psi - \phi))$$

$$+ \sin \lambda \cos 2\lambda \sin 2\theta \sin (\psi - \phi)$$

(3.15)

for P waves, and

$$R_{pp} = \sin \lambda \cos 2\delta \cos 2\theta \sin 2(\psi - \phi)$$

$$- \cos \lambda \cos \delta \cos 2\theta \cos (\psi - \phi)$$

$$+ \frac{1}{2} \cos \lambda \sin \delta \sin 2\theta \sin 2(\psi - \phi)$$

$$+ \frac{1}{2} \sin \lambda \sin 2\delta \sin 2(1 + \sin^2 (\psi - \phi)),$$

(3.16)

$$R_{ps} = \cos \lambda \cos \delta \cos \theta \sin (\psi - \phi)$$

$$+ \cos \lambda \sin \delta \sin \theta \cos 2(\psi - \phi)$$

$$+ \sin \lambda \cos 2\delta \cos \theta \cos (\psi - \phi)$$

$$- \frac{1}{2} \sin \lambda \sin 2\delta \sin 2(\psi - \phi),$$

(3.17)

for S waves, where $\sqrt{R_{pp}^2 + R_{ps}^2}$ is taken as $r_j$. The theoretical radiation patterns are finally averaged for all P- and S-wave amplitude data, as follows.

$$R^{av} = \frac{1}{n} \sum_{j} r_j,$$

(3.18)

for S waves, where $\sqrt{R_{pp}^2 + R_{ps}^2}$ is taken as $r_j$. The theoretical radiation patterns are finally averaged for all P- and S-wave amplitude data, as follows.

The obtained averages for the P and S waves are 0.43 and 0.63, respectively. These values are used for $R^{00}$ in the $Q$ tomography. By integration of Equation (3.15) or Equations (3.16) and (3.17) within $60^\circ \leq \theta \leq 120^\circ$ and $0^\circ \leq \phi \leq 360^\circ$, Boore and Boatwright (1984) obtained regional averages of 0.44 and 0.57. The present values are therefore in good agreement with previous estimates.

The validity of the averaged radiation pattern is checked using a redefined radiation coefficient:

$$R^{av} = R^{av} + \alpha (r_j - R^{av}),$$

(3.19)

where the parameter $\alpha$ varies from 0 to 0.5, and $r_j$ correspond to the averaged and theoretical radiation patterns, respectively. The root mean square (rms) residuals of $U$, and $Q$, tomography are plotted for various values of $\alpha$ in Fig. 9. These results indicates that $\alpha = 0$ (the averaged radiation pattern) gives the best result.

The radiation pattern has not been considered in many studies of seismic intensity data (e.g., Hashida and Shimazaki, 1984; Hashida, 1989; Nakamura et al., 1994). The notable exceptions
are Nakamura and Uetake (2002), who used a constant radiation coefficient proposed by Boore and Boatwright (1984), and Tsumura et al. (2000), who used the averaged radiation pattern for stations close to the nodal planes.

3.4 Ray tracing and velocity model

3.4.1 Ray equations

It is essential in tomographic studies to be able to compute travel times and ray paths in a 3D heterogeneous model fast and accurately. Among the ray tracing methods proposed by Jacob (1970), Wesson (1971) and Um and Thurber (1987), the pseudo-bending tracing method of Koketsu and Sekine (1998) is adopted in the present inversion as a computational efficient and stable technique.

In this method, seismic rays are governed by the following simultaneous ordinary differential equations defined in orthogonal curvilinear coordinates $q$:

\[
\frac{dq_i}{d\lambda} = \frac{1}{s} \frac{p_i}{h_i^2}, \quad i = 1, \ldots, n \tag{3.20}
\]

\[
\frac{dp_i}{d\lambda} = \frac{\partial s}{\partial q_i} + \frac{1}{s} \sum_{j=1}^{n} p_j \frac{\partial h_j}{\partial q_i}, \quad i = 1, \ldots, n \tag{3.21}
\]

where $h_i$ is the scaling factor of a coordinate, $\lambda$ is the ray length (e.g. Comer, 1984; Červený, 1987), $s$ is the slowness defined as inverse of velocity in the medium $(q)$, and the auxiliary variables $p_i$ must explicitly satisfy the condition

\[
\sum_{i} (p_i / h_i)^2 - s^2 = 0. \tag{3.22}
\]

The position vector is denoted $q$, and the slowness vector $p$ is defined from the condition (3.22) as

\[
p = \sum_{i} \frac{p_i}{h_i} e_i, \tag{3.23}
\]

such that $p$ is normal to a wavefront determined by a constant ray eikonal (e.g. Červený 1987). If $q$ is used, $p$ and the formulas for $e$, are as follows.

\[
e_i = \frac{1}{h_i} \frac{\partial q_i}{\partial q}, \quad \frac{\partial e_i}{\partial q_j} = \sum_{k} \Gamma_{ki}^{j} e_k, \tag{3.24}
\]

where $\Gamma_{ki}^{j}$ is Christoffel’s symbol. Equations (3.20) and (3.21) then reduce to very simple vector equations:

\[
\frac{dq}{d\lambda} = \frac{p}{s} \frac{dp}{d\lambda} = \nabla s \tag{3.25}
\]

for any orthogonal curvilinear coordinate system.

From differential geometry, the tangential and normal unit vectors $t$ and $n$ of a ray are given as

\[
t = \frac{dq}{d\lambda}, \quad n = \frac{dt}{d\lambda} \left/ \sqrt{\frac{dt}{d\lambda}} \right., \tag{3.26}
\]

at the position $Q$ indicated by the vector $q$ (e.g. Ben-Menahem & Singh 1980). From Equations (3.25) and (3.26), we have

\[
\nabla s = \frac{ds}{d\lambda} + \frac{dt}{d\lambda} n. \tag{3.27}
\]

This means that $\nabla s$ is a linear combination of $t$ and $n$, and is constrained to a single plane with $t$ and $n$.

Therefore, if $t$ and $\nabla s$ are known, the direction of $n$ can be obtained by subtracting the component parallel to $t$ from $\nabla s$. The exact relation

\[
\frac{dt}{d\lambda} = \frac{1}{s} [\nabla s - (\nabla s \cdot t)t] = \frac{1}{v} [\nabla v - (\nabla v \cdot t)t] \tag{3.28}
\]

can also be derived, which represents the procedure to obtain the direction of $n$. Subtraction of the component parallel to $t$ from $\nabla s$ gives a vector antiparallel to $n$ or parallel to the antinormal vector $m$ for any orthogonal curvilinear coordinate system.

3.4.2 Pseudo-bending method

As the results above are apparently identical to those derived by Um and Thurber (1987) for Cartesian coordinates, the procedures of Um and Thurber are also followed for seismic ray tracing in spherical coordinates. The ray path is first discretized into $N$ points at $q_1, q_2, \ldots, q_N$ in a similar manner to the bending method. The three points $q_{k-1}, q_k$ and $q_{k+1}$ are then chosen for temporarily fixing the two side points, and the central point $q_k$ is relocated so as to minimize the travel time along this segment, as shown in Fig. 10. The ray segment between $q_{k-1}$ and $q_{k+1}$ is approximated by a straight line before relocation, and is bent by moving the central point $q_k$ along $m$ away from the midpoint $q_{\text{mid}}$ of the line determined from $\nabla v$ at $q_{\text{mid}}$ and the approximation for $t$, as follows.

\[
t = \frac{1}{2L} (r_{k+1} - r_{k-1}, r_{\text{mid}}(\theta_{k+1} - \phi_{k-1}), \tag{3.29}
\]

\[
r_{\text{mid}} \sin \theta_{\text{mid}}(\phi_{k+1} - \phi_{k-1}).
\]

The distance to be moved ($R$) is determined according to Fermat’s principle by minimizing the travel time $T$ along the ray segment between $q_{k-1}$ and $q_{k+1}$.
The smoothness constraints can be summarized by the following matrix equation.

\[ D \boldsymbol{m} = \boldsymbol{d}, \]

(3.35)

where the elements of the matrix \( D \) are 4, -1 or 0, and the vector \( \boldsymbol{m} \) consists of \( -4Q_{ij}^{\text{r}} + (Q_{ij}^{\text{n}} + Q_{ij}^{\text{r}})^2 + (Q_{ij}^{\text{n}})^2 + (Q_{ij}^{\text{r}})^2 \). By combining Equations (3.11) and (3.35), the total system of equations to be solved then yields

\[ \begin{bmatrix} G \\ D \end{bmatrix} \boldsymbol{m} = \begin{bmatrix} d \\ m_0 \end{bmatrix}. \]

(3.36)

### 3.5 Inversion scheme

#### 3.5.1 Smoothness constraints

Some model regularization is often required in order to solve large-scale under-determined problems such as that considered in this study. Smoothness constraints are applied here to the \( Q \) structure by minimizing a measure of the roughness of the structure. The two-dimensional Laplacian operator \( \nabla^2 \) is adopted for this measure. The discrete representation of this operator is

\[ 4Q_{ij}^{\text{r}} - (Q_{ij}^{\text{r}} + Q_{ij}^{\text{n}} + Q_{ij}^{\text{r}} + Q_{ij}^{\text{n}}) \]

(3.33)

for grid modeling. The smoothness constraint can thus be written

\[ 4Q_{ij}^{\text{r}} - (\Delta Q_{ij}^{\text{r}} + 2Q_{ij}^{\text{n}} + 2Q_{ij}^{\text{r}}) = -4Q_{ij}^{\text{n}} - (Q_{ij}^{\text{n}} + Q_{ij}^{\text{r}} + Q_{ij}^{\text{n}} + Q_{ij}^{\text{r}}). \]

(3.34)

The smoothness constraints can be summarized by the following matrix equation.

\[ D \boldsymbol{m} = \boldsymbol{d}, \]

(3.35)

where the elements of the matrix \( D \) are 4, -1 or 0, and the vector \( \boldsymbol{m} \) consists of \( -4Q_{ij}^{\text{r}} + (Q_{ij}^{\text{n}} + Q_{ij}^{\text{r}})^2 + (Q_{ij}^{\text{n}})^2 + (Q_{ij}^{\text{r}})^2 \). By combining Equations (3.11) and (3.35), the total system of equations to be solved then yields

\[ \begin{bmatrix} G \\ D \end{bmatrix} \boldsymbol{m} = \begin{bmatrix} d \\ m_0 \end{bmatrix}. \]

(3.36)

#### 3.5.2 Trade-off between source and site effects

Since the source effect \( S \) and site effect \( t_0 \) are included in Equation (3.10) in the same manner, there is an undetermined degree of freedom in this observation equation (Andrews, 1986). In other words, if \( \Delta \ln S^* \) and \( \Delta \ln t_0^* \) are solutions of Equation (3.10), \( \Delta \ln S^* + a \) and \( \Delta \ln t_0^* + a \) are also solutions with arbitrary \( a \). Another constraint is thus required to avoid this trade-off between the source and site effects.

As the free surface of an elastic half-space doubles an input motion, the site effect must be set at 2. If local geology under
Fig. 11 P-wave velocity structure at a depth of 10 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.

Fig. 12 P-wave velocity structure at a depth of 25 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.

Fig. 13 P-wave velocity structure at a depth of 40 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.

Fig. 14 P-wave velocity structure at a depth of 65 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.
Fig. 15 S-wave velocity structure at a depth of 10 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.

Fig. 16 S-wave velocity structure at a depth of 25 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.

Fig. 17 S-wave velocity structure at a depth of 40 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.

Fig. 18 S-wave velocity structure at a depth of 65 km by Yoshii et al. (2001). Red and blue zones correspond to low and high velocities, respectively.
the station further amplifies the input motion, the effect must be greater than 2. Therefore, the inversions are performed using the following inequality constraint (Iwata and Irikura, 1988).

\[ G_i > 2. \]  

(3.37)

The constraint is introduced into the iterative solution of Equation (3.36) in the simple manner proposed by Herman (1980) for computerized tomography. If \( G_i = G_0 + \Delta G_i \) is greater than 2, where \( G_0 \) is an estimate of \( G_i \) in the previous iteration, it is sufficient to simply set \( \Delta G_i \) to 2 - \( G_0 \).

### 3.5.3 LSQR algorithm

Equation (3.36) is solved by a least-squares approach on each iteration. As Equation (3.36) is a large system of simultaneous equations, the solution is also obtained iteratively. For this inner iteration, a gradient method called the LSQR algorithm is adopted. The LSQR algorithm was developed by Paige and Saunders (1982) and first introduced into seismology by Nolet (1985) to solve a tomographic problem. The algorithm has subsequently been used by many researchers in seismic tomography. For convenience of explaining the LSQR algorithm, Equation (3.36) is rewritten as

\[ A m = b, \]  

(3.38)

where \( A = [G, D] \) and \( b = [d, m] \). Equation (3.38) is further rewritten as

\[
\begin{bmatrix}
A \\
\lambda I
\end{bmatrix}
\begin{bmatrix}
m \\
0
\end{bmatrix} = 
\begin{bmatrix}
b \\
0
\end{bmatrix},
\]  

(3.39)

by introducing the concept of damped least-squares (e.g. Aki and Lee, 1976) with a damping weight \( \lambda \). The solution of Equation (3.39) satisfies the system

\[
\begin{bmatrix}
I & A \\
A^T - \lambda^2 I
\end{bmatrix}
\begin{bmatrix}
r \\
m
\end{bmatrix} = 
\begin{bmatrix}
b \\
0
\end{bmatrix},
\]  

(3.40)

where \( r \) is the residual vector \( b - A m \). Since the coefficient matrix of Equation (3.40) is symmetric, the Lanczos process (Lanczos, 1950) is applied to reduce the matrix to bidiagonal form. The conventional QR factorization of the resultant matrix then leads to simple recursive equations for \( m \). The details of the LSQR algorithm and its advantages over other methods can be found in Paige and Saunders (1982) and van der Sluis and van der Vorst (1987).

### 3.5.4 Resolution analysis

The resolution of the inverse problem represented by Equation (3.38) is defined by the matrix

\[ R = A^* A \]  

(3.41)

where \( A^* \) is the generalized inverse of \( A \) (e.g. Menke, 1984). For the damped least-squares inversion (3.39), the generalized inverse can be calculated from

\[ A^* = (A^T A + \lambda^2 I)^{-1} A^T. \]  

(3.42)

\( R = I \) indicates that each model parameter is determined uniquely. The closeness of \( R \) to an identity matrix \( I \) therefore acts as an index of the resolving power of the inverse problem. However, \( R \) cannot be calculated from results of the LSQR algorithm because iterative methods like LSQR do not generate the generalized inverse as an intermediate product. Although Yao et al. (1999) proposed a revision of the LSQR algorithm for calculation of \( R \), the revised LSQR algorithm lacks many of the advantages of computational efficiency over full matrix inversion methods.

Instead of calculating \( R \), a checkerboard resolution test is introduced for the present inversion. The basic concept was originally proposed by Humphreys and Clayton (1988), and has subsequently been applied in travel-time tomography by Grund (1987), Inoue et al. (1990), Zhao et al. (1992a, 1992b) and many others. In the test, a checkerboard is generated by assigning positive and negative perturbations to the grid points of a homogeneous velocity model at a given interval. Synthetic amplitudes are calculated for the actual set of earthquakes and stations in the checkerboard model, and the tomographic inversion of these amplitudes is then carried out. Since the checkerboard pattern is straightforward and easy to remember, the quality of resolution can be judged readily simply by examining the image recovered by the synthetic inversion.

### 3.5.5 Initial \( Q \) model

As the least-squares problem (3.3) is solved iteratively, an initial model of the \( Q \) structure is required. This initial model is important because an inadequate model may lead to instability in the inversion. The initial model is constructed in this study in reference to the techniques used by Umino and Hasegawa (1984), Hashida (1989), Sekiguchi (1991), Tsumura et al. (2000) and many others.

In addition to the velocity structure, Zhao et al. (1992a) determined the topography of the upper boundary of the subducting Pacific Plate slab. This topography is introduced first, assuming \( Q = 1500 \) in the subducting slab for both the \( Q \) and \( Q \) structures, as shown in Fig. 19, and \( Q = 500 \) in other parts of the structures (crust and mantle above the slab). The topography of the Moho and Conrad discontinuities determined by Zhao et al. (1992b) are also included.

---

**Fig. 19** Schematic illustration of initial \( Q \) model for attenuation tomography of data for the Japanese islands.
4. 3D attenuation structures

This chapter presents the results of tomographic inversion using the method described in chapter 3 and the data described in chapter 2 for determination of the attenuation structure beneath the Japanese islands. Since the pioneering work of Backus and Gilbert (1968), seismologists have commonly recognized that the solution of an inverse problem such as seismic tomography must be compared with its resolving kernel. However, as the resolving kernel of the present inversion cannot be shown due to use of the LSQR algorithm, the checkerboard resolution tests proposed by Humphreys and Clayton (1988) are shown for qualification of the results.

Tomographic images of the $Q_s$ and $Q_v$ structures beneath the Japanese islands are expected to clarify the following issues: (1) The distribution of low-$Q_s$ zones beneath the volcanic front (2) The detection of subducting Philippine Sea Plate beneath southwestern Japan by attenuation tomography (3) The differences between $Q_s$ and $Q_v$ images, and the velocity structure.

4.1 Results of resolution analyses

Resolution analysis was performed using the checkerboard resolution test presented in section 3.5.4. Perturbations of ±50% in $Q^+$ are alternately assigned to grid points at an interval of 1\degree in the homogeneous model of $Q = 500$. Synthetic amplitudes are calculated using the actual set of earthquakes and stations for the 5 Hz frequency band. The recovered checkerboard patterns of $Q_s$ are shown in Figs. 20-29, and those for $Q_v$ are shown in Figs. 30-39. The darkest red and blue in the figures indicate perturbations of -50% and +50% or greater.

The checkerboard pattern is recovered well at depths shallower than 50 km in both the $Q_s$ and $Q_v$ results. The recovery of the $Q_s$ pattern is somewhat better than that for $Q_v$ due mainly to the greater number of S-wave rays than P-wave rays in the shallow part of the model (see the hypocenter distributions in Figs. 2 and 3. The resolution in northern Hokkaido and the northern part of southwestern Japan is not as good as in other areas due to the sparsity of earthquakes in these regions.

Compared to the results for depths of 10 and 40 km, the resolution at 25 km is rather poor. The reason for this low resolution is again the lack of a sufficient number of earthquakes at this depth due to the low seismicity in the lower crust. In addition, as most rays crossing this depth range are subvertical, the residence time of rays is quite short, limiting their contribution to $Q_s$ tomography. Although the ray orientation at 10 km is similar to that at 25 km, the high seismicity of the upper crust allows good resolution to be achieved.

At depths greater than 60 km, the resolution of $Q_v$ is better than that of $Q_s$ because P waves represent the main contribution to the maximum amplitudes of vertical ground motion due to deep earthquakes (see section 2.2). The resolution is poor in the deeper region of southwestern Japan, attributable to the rarity of deep earthquakes except for those related to the subduction beneath southern Kyushu. The JMA stations are distributed according to the extension of the Japanese islands from northeast to southwest, as shown in Fig. 1. Accordingly, most long rays, which reach the deeper region, travel northeast to southwest, and relatively few are oriented northwest to southeast. This biased distribution results in diagonal stretching of the checkerboard pattern seen in the figures.

The recovery of the checkerboard pattern at depths greater than 150 km is worse than at 150 km, rendering it necessary to mark the results of $Q$ tomography at these depths unreliable. The number of earthquakes with 2 Hz responses is about half that for the 5 Hz band. The resolution of $Q$ tomography in the 2 Hz band is thus reduced according to the availability of data.

4.2 Results of $Q$ tomography

4.2.1 Overall features

The results of $Q$ tomography are shown below as a series of figures mapping the obtained $Q$ distributions. The figures plot grid points for which adjacent cubes are intersected by five or more rays. Zones of $Q$ smaller than 150 or larger than 1500 are rendered in red or blue, respectively, and green is used to denote the initial value of 500 for the inversions. $Q$ values between 150 and 1500 are marked according to a continuum between red and blue such that warm colors indicate low $Q$ and cool colors indicate high $Q$. Solid triangles in the figures denote the locations of Quaternary volcanoes.

Figs. 40-49 and Figs. 50-59 show the $Q_s$ and $Q_v$ distributions for the 5 Hz band at depths of 10, 25, 40, 65, 90 km and every 30 km to 240 km. The cool colors are dominant in the $Q_v$ distributions compared with the $Q_s$ distribution, indicating that $Q_v$ is generally larger than $Q_s$. The ratio $Q_v/Q_s$ varies point-by-point in the study area, but is for the most part smaller than 9/4, which is the theoretically derived value for intrinsic attenuation on the assumption of no dissipation in compressional or dilatational processes (e.g. Uda, 1999).

The $Q_v$ distribution at a depth of 60 km (Fig. 43) is dominated by warm colors, and green and cool colors can only be found in the $Q_v$ distribution (Fig. 53). This implies that $Q/Q_v$ is smaller than one at that depth beneath the Japanese islands, attributable to scattering attenuation where $Q_v$ can be expected to be smaller than $Q$ (e.g. Sato and Fehler, 1997).

The $Q_s$ distributions in the 2 Hz band are shown in Figs. 60-69. As the details are not recovered well due to the small number of 2 Hz amplitude data, the distributions appears flat. However, if the result is compared with the $Q_v$ distributions for the 5 Hz band at the same depth, the general trends can be seen to be similar except for the absence of low-$Q$ zones in the 2 Hz distribution. The averaged $Q_v$ for the 2 Hz band is thus smaller than that for the 5 Hz band, which is consistent with observations of frequency-dependent $Q$ such as $Q \propto f^α (α > 0)$ at frequencies higher than 1 Hz (e.g. Sato and Fehler, 1997).

4.2.2 Tohoku region

The Tohoku region is located in a typical subduction zone, where the Pacific Plate subducts beneath northern Honshu in the
Japan Trench. Related to this subduction, many active volcanoes, such as Mts. Osore, Iwate, Kurikoma, Zao and Bandai, emerged in the Quaternary period to form a volcanic front parallel to the Japan Trench. This volcanic front is located just above the 110 km isodepth contour of the subducting slab of the Pacific Plate.

Clear low-\(Q\) zones can be recognized along the volcanic front in the \(Q\) distributions for the 5 Hz band at 40 km or shallower (Figs. 50-52). Similar low-\(Q\) zones can be identified along the volcanic front in the \(Q\) distributions at the same depths (Figs. 40-42), seen in green in the \(Q\) distributions at 10 and 25 km due to the high \(Q/\dot{Q}\) ratio in the crust.

In the crust (10 and 25 km), these low-\(Q\) zones appear only below individual volcanoes, whereas the zones extend continuously along the volcanic front at a depth of 40 km. This property can be seen more clearly in vertical profiles of the \(Q\) distribution along the four lines in Fig. 70. Among the first three lines in the direction of subduction, the lines A-A' and C-C' extend through Mts. Iwate and Bandai, respectively, whereas there are no volcanoes on the line B-B'. The line D-D' represents a part of the volcanic front in the Tohoku region. The low-\(Q\) zones beneath the volcanic front spread from the surface to a depth of 40 km in the profiles A-A' and C-C' (Figs. 71 and 73), but no low-\(Q\) zone exists in the crust immediately below the front along the B-B' profile (Fig. 72). The profile D-D' Fig. 74 shows this property well. In the \(Q\) distributions for the 2 Hz band, the continuous low-\(Q\) zone extends to a depth of 65 km, and the imaging of this zone at 65 km is clearer than at 40 km.

The features of the Tohoku region mentioned above can also be recognized in the velocity models of Yoshii et al. (2001). In those models, a continuous low-velocity zone can be seen to stretch along the volcanic front in the Tohoku region, as shown in the \(V_s\) and \(V_p\) models at a depth of 40 km (Figs. 13 and 17). This zone extends deeper in the west, but such stretching is not obvious in the present \(Q\) profiles (Figs. 71-73). The low-velocity spots in the crust immediately below the volcanoes are also seen in the \(V_s\) model at 10 km (Fig. 11), but are not significant in the \(V_p\) model (Figs. 15-18).

The low-\(Q\) zones along the volcanic front are neighbored by distinct high-\(Q\) areas in the eastern crust. These areas are more distinct in the \(Q\) distributions (Figs. 50 and 51), and almost coincide with the strata of 100 Ma or older in the Kitakami and Abukuma mountains. However, the velocity distributions in Figs. 11-18 do not exhibit such high-velocity areas to the east of the volcanic front. This disagreement may reflect the differences between the elastic and inelastic properties of the strata.

4.2.3 Kanto and Chubu regions

A low-\(Q\) area can be seen at a depth of 40 km below the Kanto region in central Japan (Fig. 52). The lowest \(Q\) values appear in the eastern part of Kanto, extending to the west. This low-\(Q\) distribution is not seen in either the \(Q_s\) distribution at the same depth (Fig. 42). Kamiya and Kobayashi (2000) identified low-velocity materials with Poisson's ratios larger than 0.3 in this region, assigned serpentine on the Philippine Sea slab based on the high Poisson's ratio and low seismicity. As hydrated materials such as serpentine attenuate only S waves, not P waves, the low-\(Q\) zone resolved here agrees with the model of a serpentinized wedge mantle identified by Kamiya and Kobayashi (2000). In support of this assignment, Winkler and Nur (1979) measured the \(Q\) values of saturated and dry rocks and found \(Q\) to be largely invariant, whereas \(Q_s\) varied clearly.

A distinct low-\(Q\) area at a depth of 65 km is apparent below the Chubu region in central Japan (Fig. 43). This area should correspond to the low-\(Q\) area found by Sekiguchi (1991) and Obara and Sato (1995) at almost the same location. Since there is no distinct low-\(Q\) zone in this area (Fig. 53), the attenuation mechanism in this region may differ from that below the volcanic front in northeastern Japan. Yoshimoto et al. (1993) observed that \(Q_s\) is larger than \(Q\) in a zone of strong scattering. Scattering attenuation may therefore be assumed in this area, for example.

4.2.4 Philippine Sea slab

In the \(Q\) and \(Q_s\) distributions at depths of 25 and 40 km in southwestern Japan (Figs. 41-42 and Figs. 51-52), high-\(Q\) zones extend continuously along the southern coasts of the Japanese islands. These areas should correspond to the subducting slab of the Philippine Sea Plate. The upper boundary of the Philippine Sea slab determined seismically by Yamazaki and Ooida (1985) and Ishida (1992) is thus used to delineate this region. The high-\(Q\) zones at depths almost coincide with these seismicity contours. In particular, the sharp corners of the contours beneath Aichi Prefecture and the western part of the Shikoku region are recovered well in the high-\(Q\) zones.

An image of the Philippine Sea slab is more distinct in the \(Q\) distributions (Figs. 41-42) than in the \(Q_s\) distributions (Figs. 51-52). From Fig. 41, the slab can be estimated to have a horizontal thickness of less than 100 km and to consist of materials with \(Q\) of 1000 or greater. The features of recovered images relating to the Philippine Sea slab can be seen clearly in vertical profiles of the \(Q\) distribution along the line E-E' (Fig. 75). This line is defined so as to follow the subduction of the slab beneath Shikoku and northern Kyushu. The profile (Fig. 76) indicates that the high-\(Q\) zone corresponding to the Philippine Sea slab does not extend beyond a latitude of 34.2°N. This northern limit of the high-\(Q\) zone agrees with the results of velocity tomography by Yoshii et al. (2001; Figs. 11-18). P-wave velocity tomography by Yamane et al. (2001), however, suggested extension into the Chugoku region. The upper boundary of the slab determined from seismicity (dashed line in the figure) corresponds well with the upper boundary of the high-\(Q\) zone.

4.2.5 Kyushu region

The line F-F' (Fig. 75) is defined to resolve the subduction of the Philippine Sea slab beneath northern Kyushu. The vertical profile of the \(Q_s\) distribution along this line (Fig. 77) shows that
the high-$Q_z$ zone reaches depths greater than 100 km, coinciding with the slab boundary determined by the seismicity. A weak low-$Q$ zone related to the volcanic front in Kyushu also appears in the crust around a longitude of 131°E.

This volcanic front consists of the Mts. Kirishima, Sakurajima and Aso volcanoes among others. Asamori and Zhao (2001) identified low-velocity zones beneath this volcanic front. At almost the same locations, low-$Q$ zones are recovered in the $Q_z$ distributions for both the 5 Hz (Figs. 50-53) and 2 Hz bands (Figs. 60-62). These zones can also be seen in the $Q_p$ distributions (Figs. 40-Fig. 43), but appear in green and are not obvious because of the large $Q/Q_p$ ratio in the crust.

### 4.2.6 Other topics

Low $Q$ is often obtained in areas of high seismicity. For example, intensive low-$Q_z$ and -$Q_p$ zones have been resolved at certain depths in the aftershock area of the 1995 Kobe earthquake and the area of the earthquake swarm induced by the 2000 Miyakejima eruption. As these areas must be highly fractured, the $Q$ value should be strongly suppressed. However, it can be noted that an inhomogeneous ray distribution may also generate virtual images of low-$Q$ zones. This inhomogeneity is avoided as far as possible by preparation of the datasets, but the effect could not be completely eliminated.

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**Fig. 20** Result of checkerboard resolution test for $Q_p$ at a depth of 10 km.

**Fig. 21** Result of checkerboard resolution test for $Q_p$ at a depth of 25 km.
Fig. 22 Result of checkerboard resolution test for $Q_p$ at a depth of 40 km.

Fig. 23 Result of checkerboard resolution test for $Q_p$ at a depth of 65 km.

Fig. 24 Result of checkerboard resolution test for $Q_p$ at a depth of 90 km.

Fig. 25 Result of checkerboard resolution test for $Q_p$ at a depth of 120 km.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands—S. SEKINE

Fig. 26 Result of checkerboard resolution test for $Q_p$ at a depth of 150 km.

Fig. 27 Result of checkerboard resolution test for $Q_p$ at a depth of 180 km.

Fig. 28 Result of checkerboard resolution test for $Q_p$ at a depth of 210 km.

Fig. 29 Result of checkerboard resolution test for $Q_p$ at a depth of 240 km.
Fig. 30 Result of checkerboard resolution test for $Q_s$ at a depth of 10 km.

Fig. 31 Result of checkerboard resolution test for $Q_s$ at a depth of 25 km.

Fig. 32 Result of checkerboard resolution test for $Q_s$ at a depth of 40 km.

Fig. 33 Result of checkerboard resolution test for $Q_s$ at a depth of 65 km.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands — S. SEKINE

Fig. 34 Result of checkerboard resolution test for $Q_s$ at a depth of 90 km.

Fig. 35 Result of checkerboard resolution test for $Q_s$ at a depth of 120 km.

Fig. 36 Result of checkerboard resolution test for $Q_s$ at a depth of 150 km.

Fig. 37 Result of checkerboard resolution test for $Q_s$ at a depth of 180 km.
Fig. 38 Result of checkerboard resolution test for $Q_s$ at a depth of 210 km.

Fig. 39 Result of checkerboard resolution test for $Q_s$ at a depth of 240 km.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands — S. SEKINE

Fig. 40  $Q_p$ structure at a depth of 10 km for the 5 Hz band.

Fig. 41  $Q_p$ structure at a depth of 25 km for the 5 Hz band.

Fig. 42  $Q_p$ structure at a depth of 40 km for the 5 Hz band.

Fig. 43  $Q_p$ structure at a depth of 65 km for the 5 Hz band.
Fig. 44  $Q_p$ structure at a depth of 90 km for the 5 Hz band.

Fig. 45  $Q_p$ structure at a depth of 120 km for the 5 Hz band.

Fig. 46  $Q_p$ structure at a depth of 150 km for the 5 Hz band.

Fig. 47  $Q_p$ structure at a depth of 180 km for the 5 Hz band.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands — S. SEKINE

**Fig. 48** $Q_p$ structure at a depth of 210 km for the 5 Hz band.

**Fig. 49** $Q_p$ structure at a depth of 240 km for the 5 Hz band.
Fig. 50  $Q_s$ structure at a depth of 10 km for the 5 Hz band.

Fig. 51  $Q_s$ structure at a depth of 25 km for the 5 Hz band.

Fig. 52  $Q_s$ structure at a depth of 40 km for the 5 Hz band.

Fig. 53  $Q_s$ structure at a depth of 65 km for the 5 Hz band.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands — S. SEKINE

Fig. 54 $Q_s$ structure at a depth of 90 km for the 5 Hz band.

Fig. 55 $Q_s$ structure at a depth of 120 km for the 5 Hz band.

Fig. 56 $Q_s$ structure at a depth of 150 km for the 5 Hz band.

Fig. 57 $Q_s$ structure at a depth of 180 km for the 5 Hz band.
Fig. 58 $Q_s$ structure at a depth of 210 km for the 5 Hz band.

Fig. 59 $Q_s$ structure at a depth of 240 km for the 5 Hz band.
Fig. 60  $Q_s$ structure at a depth of 10 km for the 2 Hz band.

Fig. 61  $Q_s$ structure at a depth of 25 km for the 2 Hz band.

Fig. 62  $Q_s$ structure at a depth of 40 km for the 2 Hz band.

Fig. 63  $Q_s$ structure at a depth of 65 km for the 2 Hz band.
Fig. 64  $Q_s$ structure at a depth of 90 km for the 2 Hz band.

Fig. 65  $Q_s$ structure at a depth of 120 km for the 2 Hz band.

Fig. 66  $Q_s$ structure at a depth of 150 km for the 2 Hz band.

Fig. 67  $Q_s$ structure at a depth of 180 km for the 2 Hz band.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands — S. SEKINE

Fig. 68  $Q_s$ structure at a depth of 210 km for the 2 Hz band.

Fig. 69  $Q_s$ structure at a depth of 240 km for the 2 Hz band.

Fig. 70  Location map of vertical profiles for $Q_s$ distribution in the Tohoku region. Lines A-A’, B-B’ and C-C’ extend in the direction of the subducting Pacific slab, and line D-D’ crosses part of the volcanic front.

Fig. 75  Location map of vertical profiles for $Q_s$ distribution in southwestern Japan. Lines E-E’ and F-F’ extend in the direction of the subducting Philippine Sea slab in Shikoku and northern Kyushu, respectively.
Fig. 71  Vertical profile of $Q_s$ structure along line A-A' in Fig. 70.

Fig. 72  Vertical profile of $Q_s$ structure along line B-B' in Fig. 70.

Fig. 73  Vertical profile of $Q_s$ structure along line C-C' in Fig. 70.
Tomographic Inversion of Ground Motion Amplitudes for the 3-D Attenuation Structure beneath the Japanese Islands — S. SEKINE

Fig. 74 Vertical profile of $Q_s$ structure along line D-D' in Fig. 70.

Fig. 76 Vertical profile of $Q_p$ structure along line E-E' in Fig. 75. Dashed curve denotes the upper boundary of the Philippine Sea slab determined from seismicity.

Fig. 77 Vertical profile of $Q_s$ structure along line F-F' in Fig. 75. Dashed curve indicates the upper boundary of the Philippine Sea slab determined from seismicity.
4.3 Seismic moments

The seismic moments $M_s$ of the earthquakes are determined simultaneously with the $Q$ distributions according to the observation equation (3.10) and the $Q^2$ model (3.12). These seismic moments are plotted against the JMA magnitudes of the earthquakes in Fig. 78. No bias is found between the moments for the 5 Hz and 2 Hz bands, indicating that the inversions were successful.

Takemura (1990) proposed the following relations between seismic moments $M_s$ in dyne · cm and JMA magnitudes $M$ for earthquakes with $M \geq 5$ in and around Japan:

$$\log M_s = 1.17M + 17.72, \quad (4.1)$$

for inland earthquakes, and

$$\log M_s = 1.5(M + 0.2) + 16.2 \quad (M \leq 6.9), \quad (4.2)$$
$$\log M_s = 2.25M + 11.3 \quad (6.2 \leq M \leq 6.9), \quad (4.3)$$
$$\log M_s = 1.5(M - 0.2) + 16.2 \quad (M \leq 6.2), \quad (4.4)$$

for offshore subduction earthquakes. Since most of the earthquakes considered in this study have magnitudes smaller than 5, Equations (4.1) and (4.3) are extrapolated to $M = 2$ and plotted in Fig. 78. The seismic moments obtained from the inversions are distributed mostly between the plotted lines, and the average agrees well with the equations.

4.4 Site effects

The site effects $q_i$ at the stations are also determined simultaneously with the $Q$ distributions and seismic moments $M_s$. The effects derived by inversion of amplitudes for the 5 Hz band are plotted in Fig. 79. Most fall close to the lower limit of 2 given by the inequality constraints (3.37), and only a few values larger than 3 are distributed apparently at random across the Japanese islands. This also indicates that the inversions were successful.

5. Application to ground motion simulation

In the previous chapter, the $Q_s$ and $Q$ structures beneath the Japanese islands were shown to have complexity similar to the velocity structure. Since such complex attenuation structures must affect seismic ground motions, the obtained $Q$ structures are applied to ground motion simulations. The incorporation of detailed $Q$ structures will lead not only to accurate prediction of ground motions for a given earthquake, but also to verification of the $Q$ structures themselves through comparison of simulated ground motions with observations.

The observations used for comparison are introduced below. The maximum amplitude distribution in this observed data is strongly affected by the 3D $Q$ structure in the Tohoku region. The ground motion simulation is performed using a hybrid method combining pseudo-spectral and finite difference methods.
5.1 Observations

The event adopted for simulation and comparison is a magnitude 4.2 earthquake that occurred on June 7, 1994 with a focal mechanism of 139.5° strike, 18.0° dip and 55.7° rake at a depth of 101 km beneath the coastal line of Ibaraki Prefecture in the northeastern Kanto region. This event was a deep subduction earthquake, and the ensuing ground motions are thus affected by the attenuation structures related to the subducting Pacific slab and the volcanic front in the Tohoku and Kanto regions.

Figure 80 shows the observed maximum amplitudes of vertical ground velocities at JMA stations in these regions. The records from seismometers at the stations denoted by diamonds were saturated, and can be considered to have experienced the largest amplitudes. No amplitude was reported from the stations denoted by plus signs, indicating the extent of motion.

Since the iso-depth contours of the high-$Q$ Pacific Plate slab extend in a north-south direction and small attenuation is expected for seismic waves traveling in this direction, the amplitude distribution is stretched to the north and south, as shown in Fig. 80. The low-$Q$ zones beneath the volcanic front (Figs. 71-74) attenuate seismic waves as they travel across the front. Accordingly, the amplitudes are quite different on the eastern and western sides of northern Tohoku even though the stations are located at the almost same distances from the hypocenter.

As the $Q$ structures were obtained by tomographic inversion of amplitude distributions (see Fig. 80), the observed distributions should be resolvable in the synthetic amplitudes calculated by Equation 3.10 using the inverted $Q$ structures. The synthetic values for the earthquake on June 7, 1994 are shown in Fig. 81. The values recover the observed amplitude distribution quite well. However, the synthetic values from the initial $Q$ model ($Q = 1500$ in the slab, $Q = 500$ elsewhere) do not reproduce the irregular amplitude distribution observed in northern Tohoku (Fig. 82), as expected due to the omission of the low-$Q$ zones beneath the volcanic front. These simple calculations verify the $Q$ structures resolved by amplitude inversion. To verify the applicability to ground motion simulation, further numerical simulations are performed below.

5.2 Method

The pseudo-spectral method (e.g. Kosloff and Baysal, 1982) is an attractive alternative to other numerical modeling schemes, such as the finite difference or finite element methods, which have been widely used for modeling of seismic wave propagation in a heterogeneous medium. In the pseudo-spectral method, the field quantities are expanded in terms of Fourier interpolation polynomials, and spatial differentiation of the quantities is performed analytically in the wavenumber domain. This accurate spatial differentiation can reduce computer memory requirements and computation times by several orders of magnitude compared with other numerical methods. However,
as the Fourier transform requires all the quantities along a coordinate axis at the same time, large-scale inter-processor communication is necessary in a parallel computing environment for differentiation along one coordinate direction.

To overcome this difficulty, a hybrid method combining the parallel pseudo-spectral method with a finite difference method (FDM) for the z-coordinate in the staggered grid was employed, as proposed by Furumura et al. (2000). As the FDM calculation is localized, interprocessor communication is required only for the quantities at several points close to the boundaries of neighboring subdomains. To account for the vertical heterogeneity of the Earth, for which fine sampling is often required and the sparse grid spacing of the Fourier differentiation is inappropriate for the z coordinate, the FDM scheme is applied to the differentiation of z.

Seismic attenuation is implemented within the pseudospectral or FDM calculation according to Graves (1996). The $Q(x, y, z)$ distribution is generated from the obtained $Q(\psi, \lambda, h)$, and the following attenuation function is calculated:

$$A(x, y, z) = \exp \left[ \frac{-\pi f_0 \Delta t}{Q(x, y, z)} \right], \quad (5.1)$$

Fig. 82 Synthetic amplitudes of vertical ground velocities calculated from the observation equation and initial $Q$ model.

Fig. 83 Amplitude distribution of vertical ground motion simulated using the obtained $Q$ structure. Color scale is the same as that in Figs. 80-82. White circles denote the locations of stations, and dashed contours represent the upper boundary of the Pacific slab.

Fig. 84 Amplitude distribution of vertical ground motion simulated using a homogeneous $Q$ model. Color scale is the same as that in Figs. 80-82. White circles denote the locations of stations, and dashed contours represent the upper boundary of the Pacific slab.
where $f_i = 5$ or $2$ Hz and $\Delta t$ is a time interval. At each time step, the updated velocity and stress fields are multiplied by $A(x, y, z)$. Although this implementation cannot distinguish between attenuation in P and S waves, since seismic motion due to medium- and large-magnitude earthquakes is typically dominated by S-wave energy, the $Q_s$ distribution is used for $Q(x, y, z)$ in Equation (5.1).

### 5.3 Results

A 768 by 384 by 150 km volume is defined in the Tohoku and Kanto regions and discretized in intervals of 1.5 km in the $x$ and $y$ directions and 0.75 km in the $z$ direction. The north orientation is rotated counter-clockwise by 30°. The elastic and attenuation properties are assigned based on the $V_p$, $V_s$, $V_p$, $V_s$, and $Q$ structures (Figs. 50-56) for the 5 Hz band.

Computation at single precision took 18 h for the 157.5 s of simulated ground motion using 8 processors on an Origin 2000 parallel computer at the Earthquake Research Institute of the University of Tokyo. Due to the limitations of computing power, only ground motions with a predominant frequency of 1 Hz were calculated.

**Fig. 83** shows the amplitude distribution of simulated vertical ground motions. The general pattern of the distribution agrees with **Fig. 80**, and the absolute amplitudes at the white circles agree with the observed amplitudes in **Fig. 80**. The results of simulation using a homogeneous $Q$ model are shown in **Fig. 84**. Due to omission of the 3D complexity in the $Q$ structure, the amplitude distribution in **Fig. 84** does not reproduce the small observed amplitudes on the western side of northern Tohoku, which can be found in **Fig. 80**. In the Chubu region, despite being beyond the volcanic front, the amplitudes are overestimated. **Fig. 84** shows the ray focusing due to the velocity structure. This ray focusing is considered to be responsible for the deviation from **Fig. 83**.

### 6. Conclusion

The $Q_s$, $Q_p$, structures beneath the Japanese islands were shown to be as complex as the velocity structure. Tomographic inversion of amplitude data revealed clear low-$Q_s$ zones beneath the volcanic front in northeastern Japan, and a distinct high-$Q_p$ area to the east of the front. This high-$Q_p$ area coincides with strata of 100 Ma or older. The low-$Q_s$ zones are restricted to immediately below volcanoes in the upper and lower crust, but extend continuously along the volcanic front at a depth of 40 km. The $Q_p$ distribution exhibits a similar distribution, although the averaged $Q_s$ in the crust is significantly lower than the averaged $Q_p$ and the corresponding low-$Q_p$ zones are not as obvious. These tendencies can also be found along the volcanic front related to subduction of the Philippine Sea Plate.

A low-$Q_p$ area was also recognized at a depth of 40 km below the Kanto region in central Japan. Kamiya and Kobayashi (2000) also found low-velocity materials with relatively high Poisson’s ratios in this area, attributing the materials to serpentine in the Philippine Sea slab. From measurement of $Q_p$ for saturated and dry rocks, Winkler and Nur (1979) found the $Q_p$ value of saturated and dry rocks to be almost the same, whereas $Q_s$ differs clearly. $Q_s$ is therefore considered to exhibit little suppression in the low-$Q_s$ areas, reflecting the weaker attenuation of P waves by hydrated serpentine than for S waves.

A distinct low-$Q_p$ area was identified at a depth of 65 km below the Chubu region in central Japan, corresponding to the low-$Q_s$ area found by Sekiguchi (1991) and Obara and Sato (1995) in the same region. However, no low-$Q_p$ zone can be found in this area, implying a different attenuation process from that below the volcanic front in northeastern Japan.

A high-$Q_p$ area was resolved along the seismically determined upper boundary of the Philippine Sea slab. This area is more distinct in the $Q_p$ distribution (average of 1000) than in the $Q_s$ distribution. In the Shikoku region, the high-$Q_p$ area does not extend beyond a latitude of 34.2°N, and appear to descend to depth in that region. In the Kyushu region, the high-$Q_p$ area reaches a depth of 100 km or greater, coinciding with the slab boundary determined by seismicity.

As such complex attenuation structures must affect seismic ground motions, the obtained $Q$ structures were applied to ground motion simulation. The amplitude distribution was reproduced well by this simulation. Without the $Q$ distribution, however, good agreement with the observed distribution could not be obtained.

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### Reference


地動振幅トモグラフィによる日本列島の三次元減衰構造

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要 旨
地震動の速度振幅データの上下動成分を用いて、トモグラフィ解析を行なうことにより日本列島下の三次元減衰構造を、$Q$ 値を指標として求めた。減衰構造は、物質の弾性的構造や非弾性的構造を示しているという点で、速度構造と同じように複雑なものをである。本研究においては、1994年の1月から2000年の12月までのカタログデータを用い、2Hzと5Hzという二つの周期帯において三次元減衰構造（$Q_L$ および $Q_s$）を求めた。その結果として、東北日本での火山フロントに沿った低 $Q$ 値と、その東側にある高 $Q$ 値が確認された。また、関東地方の40km付近には低 $Q$ の領域が見られ、西南日本においてはフィリピン海スラブの上面に沿った部分が高 $Q$ 値であることが示された。さらに、関東で起こった地震に対して、求められた三次元 $Q$ 構造を用いて地震動シミュレーションを行なったところ、かなりの精度で観測振幅を再現できた。

キーワード：減衰構造, $Q$ 値, 振幅データ, トモグラフィ